Vektoralgebrai kiegészítések

Kifejtési tétel, többszörös szorzatok:

$$\begin{aligned}
(\boldsymbol{a}\times\boldsymbol{b})\times\boldsymbol{c} &= (\boldsymbol{ac})\boldsymbol{b}-(\boldsymbol{bc})\boldsymbol{a}\\
\boldsymbol{a}\times(\boldsymbol{b}\times\boldsymbol{c}) &= (\boldsymbol{ac})\boldsymbol{b}-(\boldsymbol{ab})\boldsymbol{c}\\
(\boldsymbol{a}\times\boldsymbol{b})(\boldsymbol{c}\times\boldsymbol{d}) &= (\boldsymbol{ac})(\boldsymbol{bd})-(\boldsymbol{bc})(\boldsymbol{ad}) \quad = \begin{vmatrix}\boldsymbol{ac} & \boldsymbol{bc}\\ \boldsymbol{ad} & \boldsymbol{bd}\end{vmatrix} \qquad \textit{(Lagrange-azonosság)}\\
\rightarrow \quad (\boldsymbol{a}\times\boldsymbol{b})^2 &= \boldsymbol{a}^2\boldsymbol{b}^2-(\boldsymbol{ab})^2 \quad \rightarrow \quad \boldsymbol{a}^2\boldsymbol{b}^2 \ge (\boldsymbol{ab})^2\\
(\boldsymbol{a}\times\boldsymbol{b})\times(\boldsymbol{c}\times\boldsymbol{d}) &= (\boldsymbol{acd})\boldsymbol{b}-(\boldsymbol{bcd})\boldsymbol{a}\\
\boldsymbol{abc}\cdot\boldsymbol{def} &= \begin{vmatrix}\boldsymbol{ad} & \boldsymbol{ae} & \boldsymbol{af}\\ \boldsymbol{bd} & \boldsymbol{be} & \boldsymbol{bf}\\ \boldsymbol{cd} & \boldsymbol{ce} & \boldsymbol{cf}\end{vmatrix}
\end{aligned}$$

Vektor vetületei

Az $\boldsymbol{a}$ vektornak $\boldsymbol{v}(\neq \boldsymbol{0})$-re vonatkozó párhuzamos és merőleges vetületei:

$\boldsymbol{v}$ *irányú (előjeles) hossz:* $\quad \dfrac{\boldsymbol{va}}{|\boldsymbol{v}|} = \boldsymbol{v}^0\boldsymbol{a}$

$\boldsymbol{v}$ *irányú vetület:* $\quad \dfrac{(\boldsymbol{va})\boldsymbol{v}}{|\boldsymbol{v}|^2} = (\boldsymbol{v}^0\boldsymbol{a})\boldsymbol{v}^0$

$\boldsymbol{v}$*-re merőleges hossz:* $\quad \dfrac{|\boldsymbol{v}\times\boldsymbol{a}|}{|\boldsymbol{v}|} = \left|\boldsymbol{v}^0\times\boldsymbol{a}\right|$

$\boldsymbol{v}$*-re merőleges vetület:* $\quad \dfrac{(\boldsymbol{v}\times\boldsymbol{a})\times\boldsymbol{v}}{|\boldsymbol{v}|^2} = (\boldsymbol{v}^0\times\boldsymbol{a})\times\boldsymbol{v}^0 = \boldsymbol{a}-(\boldsymbol{v}^0\boldsymbol{a})\boldsymbol{v}^0$

$\boldsymbol{v}^0$ *jelentése: a* $\boldsymbol{v}$ *irányú egységvektor, előfordul az* $\boldsymbol{e}_v$ *jelölés is.*

$\boldsymbol{v}^0 = \boldsymbol{e}_v = \dfrac{\boldsymbol{v}}{|\boldsymbol{v}|} \qquad (\boldsymbol{v}\neq\boldsymbol{0})$

$\boldsymbol{a}$ vektor felbontása, lineárisan független $\boldsymbol{u},\boldsymbol{v},\boldsymbol{w}$ $\quad(\boldsymbol{uvw}\neq 0)$ összetevőkre:

$$\boldsymbol{a} = \frac{\boldsymbol{a}\,\boldsymbol{vw}}{\boldsymbol{uvw}}\,\boldsymbol{u} + \frac{\boldsymbol{u}\,\boldsymbol{a}\,\boldsymbol{w}}{\boldsymbol{uvw}}\,\boldsymbol{v} + \frac{\boldsymbol{uv}\,\boldsymbol{a}}{\boldsymbol{uvw}}\,\boldsymbol{w}$$

Vektor-skalár függvények differenciálási szabályai:

$$\begin{aligned}
\boldsymbol{c}' &= 0 \\
(\boldsymbol{v} \pm \boldsymbol{w})' &= \boldsymbol{v}' \pm \boldsymbol{w}' \\
(f(t) \cdot \boldsymbol{v})' &= f'(t) \cdot \boldsymbol{v} + f(t) \cdot \boldsymbol{v}' \\
(\boldsymbol{v}\boldsymbol{w})' &= \boldsymbol{v}'\boldsymbol{w} + \boldsymbol{v}\boldsymbol{w}' \\
(\boldsymbol{v} \times \boldsymbol{w})' &= \boldsymbol{v}' \times \boldsymbol{w} + \boldsymbol{v} \times \boldsymbol{w}' \\
\frac{d\boldsymbol{v}}{dt} &= \frac{d\boldsymbol{v}}{d\tau} \cdot \frac{d\tau}{dt}
\end{aligned}$$

További szabály:

$$\frac{d\,|\boldsymbol{v}|}{dt} = \frac{\boldsymbol{v}\dot{\boldsymbol{v}}}{|\boldsymbol{v}|} = \boldsymbol{v}^0\dot{\boldsymbol{v}}$$